



# A global coal production forecast with multi-Hubbert cycle analysis

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## ABSTRACT

Based on economic and policy considerations that appear to be unconstrained by geophysics, the Intergovernmental Panel on Climate Change (IPCC) generated forty carbon production and emissions scenarios. In this paper, we develop a base-case scenario for global coal production based on the physical multi-cycle Hubbert analysis of historical production data. Areas with large resources but little production history, such as Alaska and the Russian Far East, are treated as sensitivities on top of this base-case, producing an additional 125 Gt of coal. The value of this approach is that it provides a reality check on the magnitude of carbon emissions in a business-as-usual (BAU) scenario. The resulting base-case is significantly below 36 of the 40 carbon emission scenarios from the IPCC. The global peak of coal production from existing coalfields is predicted to occur close to the year 2011. The peak coal production rate is 160 EJ/y, and the peak carbon emissions from coal burning are 4.0 Gt C (15 Gt CO<sub>2</sub>) per year. After 2011, the production rates of coal and CO<sub>2</sub> decline, reaching 1990 levels by the year 2037, and reaching 50% of the peak value in the year 2047. It is unlikely that future mines will reverse the trend predicted in this BAU scenario.

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## 1. Introduction

A quick check of press reports on July 8, 2009, yielded the following three stories:

(INDIA) Coal shortage is set to hit Nalco's aluminium production this year. A 110 MW unit of Nalco's captive power plant was shut down this week due to a shortage of coal. The drop in power generation has affected aluminium production, prompting company officials to discuss the coal supply issue with MCL (Mahanadi Coalfields Limited) authorities.<sup>1</sup>

(AUSTRALIA) Coal exports from Australia's Newcastle port, the world's largest coal export terminal, rose 9.5% in the latest week, while the number of vessels queuing off the port rose to the highest in 18 months. Trade sources said on Wednesday ship queues had risen for a second week due to unplanned maintenance at the port and as wet weather slowed production at some mines leaving some vessels short of cargo. Exports from the eastern coast port, which ships mostly thermal coal used in power generation, rose to 1.85 million tonnes in the week to July

6, port data showed. Ship queues stood at 49 as of Tuesday, according to data from the Web site of the Hunter Valley Coal Chain Logistics Team (HVCCLT), which coordinates coal movements from mines to the port. Of the 49 vessels waiting to load coal, 13 had coal availability issues while an additional seven did not have coal available when they arrived, HVCCLT said. The number of coal ship arrivals, a key indicator of demand, also rose by 4–24, while waiting time for vessels scheduled to load coal fell to 11.7 days.<sup>2</sup>

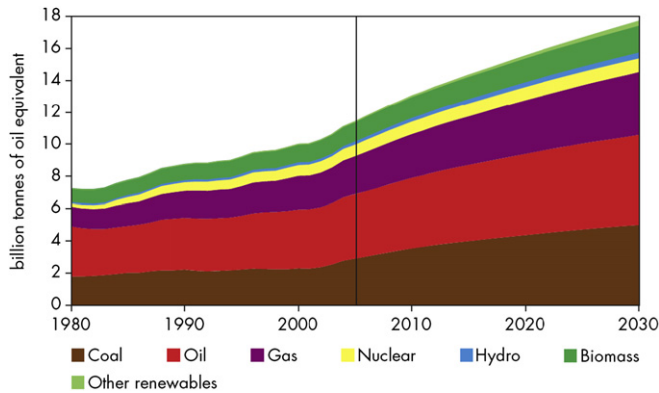
(SOUTH AFRICA) Coal volumes railed to South Africa's Richards Bay Coal Terminal are continuing to fall far short of the corridor's nameplate yearly capacity of 70 million tonnes, coming in at closer to an annualized 60 million tonnes. Mr Chris Wells acting CEO of Transnet said that South Africa will do well to achieve volumes of between 64 million tonnes and 65 million tonnes for the year as a whole. At the start of its 2009 financial year in April, the State-owned group's rail division said it had budgeted to handle 69-million tonnes up until March 31, 2010, but under-supply from the mines had all but put paid to that ambition. Rail volumes last year fell to a very disappointing 61.9 million tonnes, capping a 3-year trend of underperformance, with volumes having fallen consistently since 2006, when 68.8

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<sup>1</sup> ANGUL (Orissa): Coal crisis hits Nalco's aluminium production, 2009-07-08 14:50:00, Commodity Online, [www.commodityonline.com](http://www.commodityonline.com).

<sup>2</sup> Australia Newcastle coal exports up 9.5%, 2009-07-08, 10:59:02 AM, PERTH, Reuters.



**Fig. 1.** World primary energy demand in the Reference Scenario by the International Energy Agency (IEA) is often equated with the energy providers' ability to deliver. "Coal sees the biggest increase in demand among all primary energy sources in absolute terms between 2005 and 2030, closely followed by natural gas and oil. Coal demand jumps by 38% between 2005 and 2015 and 73% by 2030 – a faster increase than in previous editions of the Outlook." (Anonymous, 2007).

million tonnes was railed. There has been a concerted Transnet effort to improve rail's performance since September, but now the mines are running short of supply. Both sides need to reach a deal that optimizes South Africa's foreign exchange potential.<sup>3</sup>

Is it possible that global demand for coal is outstripping supply? Is production in some of the major coal-exporting countries falling? Can it be that peak of global coal production is near?

Our answer to these three questions is yes.

Faced with the imminent global peaks of oil and coal production, economists, scientists, and policy makers have been taking radically different positions. One prominent group claims that there can be no peak of production because society's ability to find new fossil fuel deposits and exploit them is practically infinite, see, e.g., [1].<sup>4</sup> Besides, some add, many of the oil and gas reservoirs are recharged with hydrocarbons in real time, e.g. [2]. This group sees an uninterrupted increase in the demand and production of fossil fuels in the foreseeable future, depicted in Fig. 1 from [1].

As a reaction to such claims, a powerful social movement has sprung up to demonize fossil fuels as the root of all pain and suffering on the planet. A central belief in this camp is that fossil fuels cause most if not all of the recent climate change.

Remarkably, both sides apparently assume an infinite capacity of the Earth to yield fossil fuels at arbitrary rates. They only differ when it comes to dealing with the effects of this ever-increasing production. In short, the action and reaction described here are merely two aspects of the same dialectic process described by Chalybäus [3] beautifully, see Sections 5 and 7.

An opposite view was best summarized by William Stanley Jevons [4]:

The expression "exhaustion of our coal mines," states the subject in the briefest form, but is sure to convey erroneous notions to those who do not reflect upon the long series of changes in our industrial condition which must result from the gradual deepening of our coal mines and the increased price of fuel. Many persons perhaps entertain a vague notion that some

day our coal seams will be found emptied to the bottom, and swept clean like a coal-cellar.

Those who adhere to this line of thinking lump coal, petroleum, and natural gas together, alleging that the cellars filled with these respective resources will be emptied soon, so they should be shut tight. Holders of this view, make parallel demands to abruptly curtail fossil fuel consumption ("energy security") and emissions ("global warming"). These demands lack consistency, for one cannot curtail that which one does not have.

An old Jew in Galicia once made an observation: "When someone is honestly 55% right, that's very good and there's no use wrangling. And if someone is 60% right, it's wonderful, it's great luck, and let them thank God. But what's to be said about 75% right? Wise people say this is suspicious. Well, and what about 100% right? Whoever says he's 100% right is a fanatic, a thug, and the worst kind of rascal." [5]. Which brings us to the last group of scientists, who do not mind being 55% right and generally disparaged. These people conceive of a spherical Earth with finite resources (M. King Hubbert and Paul R. Ehrlich) and interconnected fragile ecosystems (James Lovelock and Vaclav Smil). They also quantify how woefully inadequate and nonphysical the governing economic models are in addressing the society's future and present needs (Nicolas Georgescu-Roegen, Herman Daly, Charles Hall, and Philip Mirowski).

The authors intend to stay within the wise Jew's estimate of maximum tolerable correctness,<sup>5</sup> and set out to estimate the future of coal production in the world through the actual production data and Central Limit Theorem. We build on the three earlier papers [6–8], and use the coal production data from the Supplemental Materials to Mohr and Evans [9], who have gathered and made public a global data set that agrees with our earlier sources for China and USA. However, we have not been able to replicate several of the coal reserves estimates published by Mohr and Evans [9]; the discrepancies are listed in the [Online Supporting Materials](#).

The base-case global coal production forecast in this paper is compared with forty Intergovernmental Panel on Climate Change (IPCC) scenarios in the SRES Report [10]. Most of the IPCC scenario writers accepted the common myth of 200–400 years of coal supply, and now their "eternal" (100 years plus) growth of carbon dioxide emissions in turn is a part of the commonly accepted social myth. It seems, therefore, that the present attempt to inject some geophysics into the debate will be an uphill battle.

The literature on global coal production, CO<sub>2</sub> emissions and electric power generation is immense, and it is impossible to quote every deserving paper. Here is but a sample of the recent relevant publications: [11–17].

## 2. A multi-Hubbert cycle approach

Ever since M. King Hubbert's seminal work [18–22], there has been an ongoing controversy about the very existence of "Hubbert cycles," and their ability to predict the future rate of mining an earth resource. This controversy seems to have been more ideological than scientific. Emergence of Hubbert cycles that approximate time evolution of the total production rates from populations of coal mines, oil reservoirs, ore deposits, etc. was put to rest in the notes to a course offered at Berkeley by Patzek [23]. These arguments are summarized here as [Appendices A and B](#).

<sup>3</sup> *RBC coal exports to remain below capacity in 2009*, [steelguru.com](http://steelguru.com), 2009-07-07 (from [Miningweekly.com](http://Miningweekly.com)).

<sup>4</sup> In the 2008 version, however, the perpetual growth predictions of 2007 were toned down, see [www.worldenergyoutlook.org/docs/weo2008/WE020084-es-english.pdf](http://www.worldenergyoutlook.org/docs/weo2008/WE020084-es-english.pdf).

<sup>5</sup> This paper is falsifiable in parts and as a whole. If we are wrong, stricter controls on the increase of coal production and use will have to be applied. If we are right, major restructuring and shrinking of the global economy will follow. Because the second alternative is so much more far-reaching, prudent policy makers ought to perhaps consider it.

The emergence of Hubbert cycles is assured by the Central Limit Theorem,<sup>6</sup> provided that the coal mines in the population are numerous, independent of each other, and there are few artificial regulatory constraints on mine production. The total coal production in a country or worldwide can be treated as a sum of many independent random variables that in turn describe production from individual coalfields and/or mines. By the Central Limit Theorem, the distribution of the sum tends to be normal or Gaussian. Thus, the fundamental Hubbert curve or “cycle” is approximately symmetric and bell-shaped. The story does not end there, however. Expansions of existing mines or additions of new mines result in an oscillatory, damped deviation of true production from the fundamental Hubbert cycle. This deviation can be modeled with a Fourier series or, simply, with a few secondary Hubbert curves that capture depletion of these expansions or additions.

The authors of this paper have maintained that coal production data series are independent of reserve numbers, and tell a very different story [7,8]. One other way checking the maturity of underground coal mining is to look at mine depth, but that is a more qualitative approach. That said, in China 2 out of 3 approaches suggest a near-term peak in coal production. Coal-producing areas have two sets of numbers: (1) reserves at existing mines that are supported by engineering studies and (2) proved coal reserves that are a much broader category than proven oil reserves. Over time, these two sets of numbers trend in opposite directions. For example, in Illinois, the number two proven coal reserve holder in the U.S., coal production has declined by almost half from what it was 20 years ago. By playing such number games governments worldwide get the big reserves they want, while mine operators do not promise more than they can deliver. It is clever politics, but it leaves us analysts with the question of how to reconcile two dramatically different sets of reserve estimates. We do not put much stock in the proved reserves, but the existing mine numbers are good and should always be below (or equal to) the Hubbert curve approximations. The proved reserve numbers are the source of the myth of a 200–400 years supply of coal worldwide at the rate of production of roughly 6.5 billion tonnes per year.

There are distinct advantages to using the Hubbert curves as models of large populations of mines:

1. A single Hubbert curve is the natural approximating function for the time evolution of the total coal production rate from each fixed population of coalfields.
2. New curves can be added to describe mine expansions in any of the coalfields or production from new coalfields.
3. The Hubbert curves are based on coal production, and not on ill-defined and subjective coal “reserves.”
4. Historical production trends reflect the prevailing economics prior to the time of production.

The advantages of Hubbert curves in large-scale models of resource production make them a valuable tool in policy-making.

At a given time, a collection of Hubbert curves that fully describes the existing populations of coal mines in the known coalfields carries no information about the possible future mine populations in new coalfields. Thus, the Hubbert cycle predictions almost always underpredict the true future production rate of a resource. The underprediction of production in immature coal

provinces, such as Alaska, Kazakhstan, East Siberia, Australia and Mongolia, may be substantial, but it may take decades to produce very large quantities of coal from future mines. In the meantime, production from the existing mine populations may decline so much that no future production increases can reverse the current global trend. Because the world is only 2–5 years away from the peak of coal production, numerous major new mines would have to be brought online within half a decade to arrest the predicted decline of global coal production. This may be difficult because of the remoteness of Mongolia, or environmental and water supply concerns in India, Australia, and the United States.

To convert the mass of coal of different ranks to the corresponding higher heating values (HHVs), the following averages of the HHV are used: 30 MJ/kg of anthracite, 27 MJ/kg of bituminous coal, 21 MJ/kg of subbituminous coal, and 15 MJ/kg of lignite. These averages are then multiplied by the annual production of coal reported by rank. The mass of all coals is converted to the corresponding carbon dioxide emissions with a procedure described in Appendix C.

The best fits (or “matches”) of historic coal production are carried out by minimizing the sum of squares of the deviations between the sum of Hubbert curves and the data. As the figures below demonstrate, these matches can be quite accurate. The “linearized Hubbert cycle” approach, i.e., a linear fit of the ratio of coal production rate divided by cumulative coal production versus the cumulative coal production is avoided. This fit is unstable and often does not intercept zero to yield an estimate of ultimate coal recovery. Where stable estimates exist, the multi-cycle fits presented here yield values similar to those published by Mohr and Evans [9]. Otherwise, there can be significant differences between our ultimate coal recoveries and those by Mohr and Evans.

### 3. Global coal production and CO<sub>2</sub> emissions

The key coal-producing countries are discussed in the [Online Supporting Materials](#) to this paper. The ultimate coal production, peak production rate, ultimate CO<sub>2</sub> emissions and peak emissions rate by country are listed in Table 1. The Hubbert cycles represent mining districts (populations of mines), rather than individual mines. In the case of Mongolia, the 2105 peak production will come from mines that were not producing in 2005–2007, but these future mines are assumed to follow the early production trend. In most other cases, the mining districts operated in earnest in 2005–2007, and their predicted future development is consistent with historical records.

The findings of this report are illustrated in Figs. 2–6 and summarized here:

1. The HHV of global coal production is likely to peak in the year 2011 at 160 EJ/y, see Fig. 2.
2. The global CO<sub>2</sub> emissions from coal will also peak in 2011 at 15 Gt/yr, see Fig. 3.
3. The estimated CO<sub>2</sub> emissions from global coal production will decrease by 50% by the year 2050, see Fig. 3.
4. Between the years 2011 and 2050, the average rate of decline of CO<sub>2</sub> emissions from the peak is 2% per year, and this decline increases to 4% per year thereafter, see Fig. 6.
5. It may make sense to have carbon capture and sequestration (CCS) to alleviate the highest CO<sub>2</sub> emissions between now and the year 2020 or so.
6. Given the imminence of the global coal production peak, a better alternative would be to gradually replace the existing electrical power generation blocks with the new ultra super-critical steam blocks (steam temperatures of 620–700 °C, and pressures of 220–250 bars), whose electrical efficiency is close to 50%, compared with the ~35% efficiency currently realized. This replacement might ultimately lower current CO<sub>2</sub>

<sup>6</sup> Proposed by Abraham de Moivre who, in an article published in 1733, used the normal distribution to approximate the distribution of the number of heads resulting from many tosses of a fair coin. This finding was far ahead of its time, and was nearly forgotten until Pierre-Simon Laplace rescued it from obscurity in his *Théorie Analytique des Probabilités* published in 1812.

**Table 1**  
Summary of coal production and CO<sub>2</sub> emissions by largest coal-producing countries on the Earth.

Country	EJ peak <sup>a</sup> (year)	Ultimate coal production (EJ)	Peak coal rate (EJ/y)	Ultimate CO <sub>2</sub> emissions (Gt)	Peak CO <sub>2</sub> rate (Gt/y)
China	2011	4015.6	75.8	365.0	6.9
USA <sup>b</sup>	2015	2756.7	26.8	250.5	2.4
Australia	2042	1714.5	23.5	155.8	2.1
Germany/Poland	1987	1104.4	14.9	100.4	1.4
FSU <sup>c</sup>	1990	1070.3	20.3	97.3	1.8
India	2011	862.6	13.6	78.4	1.2
UK	1912	753.0	7.7	68.4	0.7
S. Africa	2007	478.6	6.8	43.5	0.6
Mongolia	2105	279.2	3.2	25.4	0.3
Indonesia	2012	135.5	5.8	12.3	0.5
Global ultimate/peak	2011	13,170.5	160	1197.0	15.0

<sup>a</sup> Note that sometimes the peaks of produced coal tonnes and EJ do not coincide.

<sup>b</sup> Excluding Alaskan coal

<sup>c</sup> The Former Soviet Union, excluding the Russian Far East coal.

emissions from coal-fired power stations by  $15/35 \approx 40\%$  for the same amount of electricity.

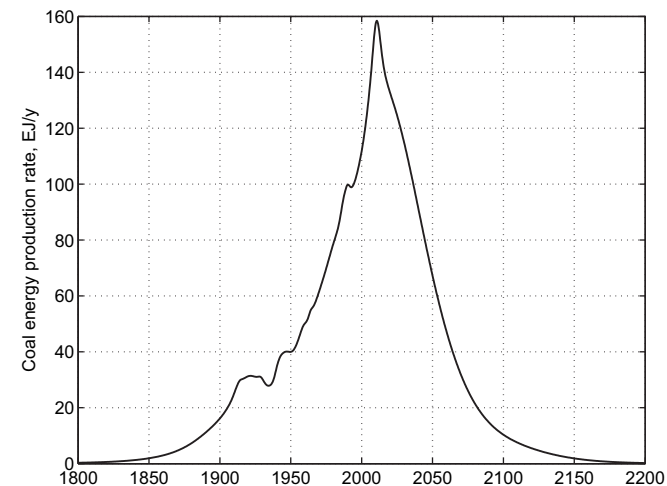
#### 4. Error analysis

As shown in Appendix A, matching cumulative production of a distributed resource with multiple Hubbert curves captures most of the variance of data. A typical example for the U.S. coal production is shown in Fig. 7. The statistics of linear fits of cumulative coal production by all major producers are summarized in Table 2. It is seen that the mean square errors of the fits are negligible and the  $R^2$  statistics are close to 1.

As shown in Appendix C, the coal heating values reported by coal producers are usually higher than the average heating values for raw coals shipped to electric power stations. Thus, the total heating value of global coal production may be exaggerated by up to, say,  $3/27 \approx 10\%$ . The same statement is true for CO<sub>2</sub> emissions from the global coals. Therefore, the calculations performed here are conservative, if anything, erring on the high side of carbon dioxide emissions.

#### 5. The IPCC coal production scenarios

Here is a brief summary of the process used by the IPCC to arrive at their fossil fuel production and greenhouse gas emissions scenarios [10]:

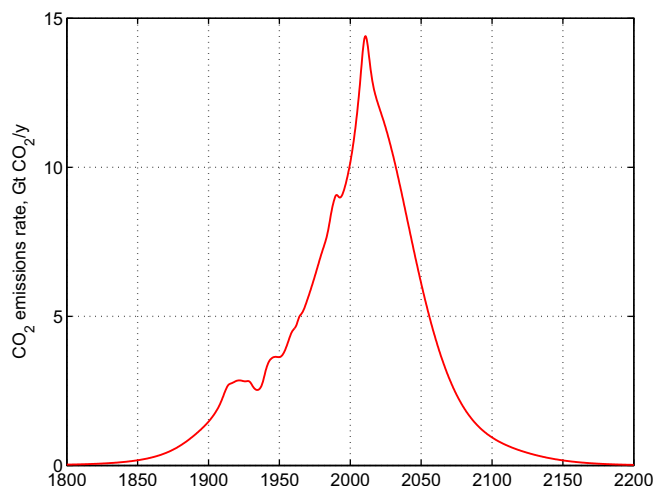


**Fig. 2.** The best multi-Hubbert cycle match of the historical rate of production of energy in coal of all ranks worldwide. The year of peak production is 2011, and peak coal energy production (higher heating value) is 160 EJ/y. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), and Supplemental Materials to (Mohr and Evans, 2009) [9].

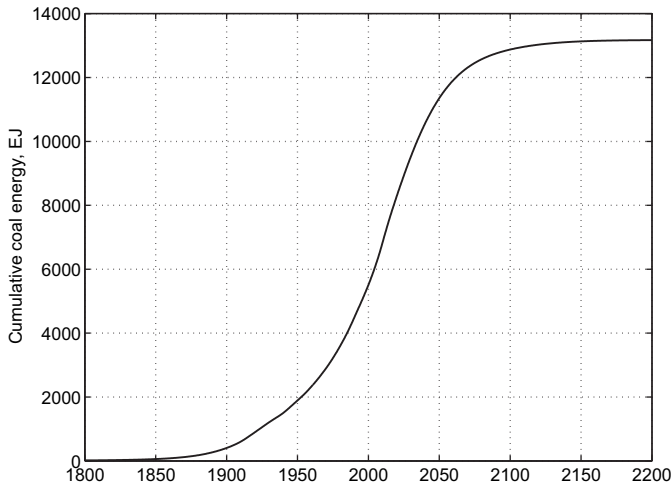
The Special Report on Emissions Scenarios (SRES) writing team included more than 50 members from 18 countries who represent a broad range of scientific disciplines, regional backgrounds, and non-governmental organizations. The team, led by Nebojsa Nakicenovic of the International Institute for Applied Systems Analysis (IIASA) in Austria, included representatives of six scenario modeling groups and lead authors from all three earlier IPCC scenario activities – the 1990 and 1992 scenarios and the 1994 scenario evaluation. The SRES preparation included six major steps:

- analysis of existing scenarios in the literature;
- analysis of major scenario characteristics, driving forces, and their relationships;
- formulation of four narrative scenario “storylines” to describe alternative futures;
- quantification of each storyline using a variety of modeling approaches;
- an “open” review process of the resultant emission scenarios and their assumptions; and
- three revisions of the scenarios and the report subsequent to the open review process, i.e., the formal IPCC Expert Review and the final combined IPCC Expert and Government Review.

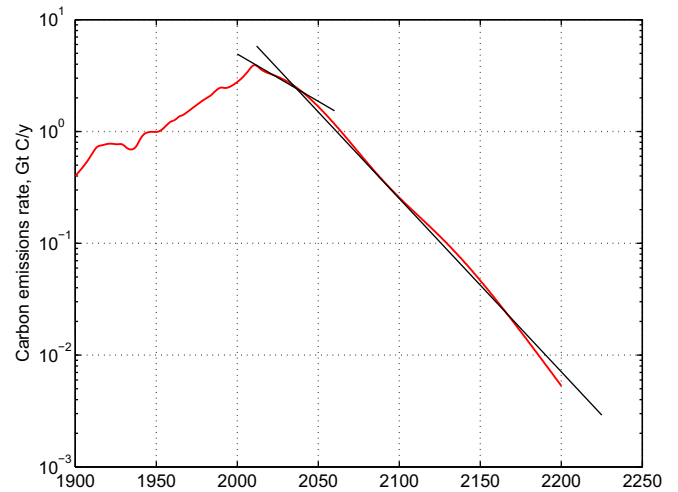
As required by the Terms of Reference, the SRES preparation process was open with no single “official” model and no exclusive “expert teams.” To this end, in 1997 the IPCC advertised in relevant



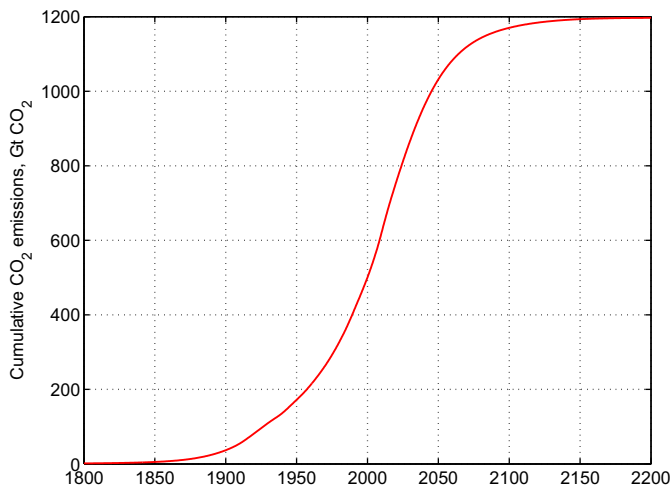
**Fig. 3.** The best multi-Hubbert cycle match of the historical CO<sub>2</sub> emissions from coal burning worldwide. The year of peak emissions is 2011, and the peak rate of emissions is 15 Gt/yr. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), and Supplemental Materials to (Mohr and Evans (2009) [9].



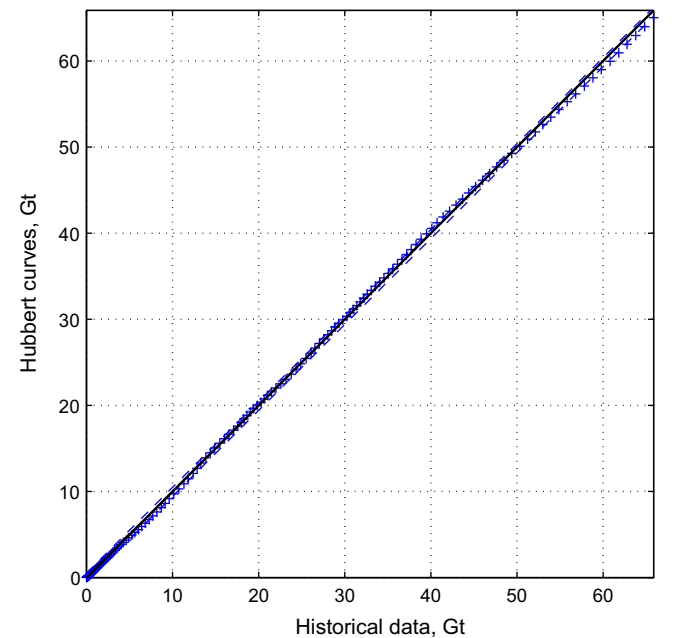
**Fig. 4.** The best multi-Hubbert cycle match of the historical cumulative production of coal of all ranks worldwide. The ultimate higher heating value of global coal production is 13,200 EJ. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), and Supplemental Materials to Mohr and Evans (2009) [9].



**Fig. 6.** A semi-logarithmic plot of carbon emissions from coal burning worldwide. The year of peak emissions is 2011, and the peak rate of emissions is 4.0 Gt C/yr. In 1990, a benchmark year for IPCC, the calculated rate of emissions was 2.4 Gt C/yr. The rates of decline from the peak are 1.9% per year until 2046, and 3.6% thereafter. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), and Supplemental Materials to Mohr and Evans (2009) [9].



**Fig. 5.** The best multi-Hubbert cycle match of the historical rate of CO<sub>2</sub> emissions worldwide. The predicted cumulative emissions are 1200 Gt CO<sub>2</sub>. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), and Supplemental Materials to Mohr and Evans (2009) [9].



**Fig. 7.** A cross-plot of cumulative coal production in the U.S. predicted with multiple Hubbert curves, versus historical data. Points along the diagonal denote perfect fit. The broken lines are 95% confidence intervals for the linear fit of the data. The  $R^2$  statistic is 0.9998, and the mean squared error (average squared residual) is 0.0694 Gt<sup>2</sup>.

scientific journals and other publications to solicit wide participation in the process. A web site documenting the SRES process and intermediate results was created to facilitate outside input. Members of the writing team also published much of their background research in the peer-reviewed literature and on web sites.

Our best production-based predictions of coal energy and carbon emissions produced worldwide are compared with the 40 IPCC scenarios released online.<sup>7</sup> The comparison is shown in Figs. 8 and 9. Between the years 1990 and 2011 all but two of the IPCC scenarios are at or below the actual world coal production and emissions. In contrast, after 2011, most of the IPCC predictions increase unrealistically in a variety of exponential ways. Thirty-six out of 40 of these scenarios deviate significantly upwards from our base-case, up to a factor of 100; see Fig. 10. In particular, 2 IPCC scenarios peak in the year 1990, 3 in 2020, 3 in 2030, 3 in 2040, 13 in

2050, while in the 16 remaining scenarios coal production simply grows exponentially until the year 2100. Because IPCC did not rank its forty scenarios on purpose, the 16 nonphysical outliers, and 4 other scenarios,<sup>8</sup> were given *de facto* a weight equal to the more realistic lowest scenarios. The policy makers tend to focus on the most extreme outcomes, and the outliers have gained prominence as inputs to the subsequent climate models. The real problem 40 years from 2009 will be an insufficient supply of fossil energy, not its overabundance, as the IPCC economists would have it.

<sup>7</sup> In Appendix SRES Version 1.1, published at [www.grida.no/publications/other/ipcc\\_sr/?src=/climate/ipcc-emission/101.htm](http://www.grida.no/publications/other/ipcc_sr/?src=/climate/ipcc-emission/101.htm), and accessed on July 4, 2009.

<sup>8</sup> Twenty out of the 40 IPCC scenarios result in carbon emissions in the year 2100 that are 20–100 times the base-case here, see Fig. 10.

**Table 2**  
Summary of fit statistics of the cumulative coal production by country with multiple Hubbert curves.

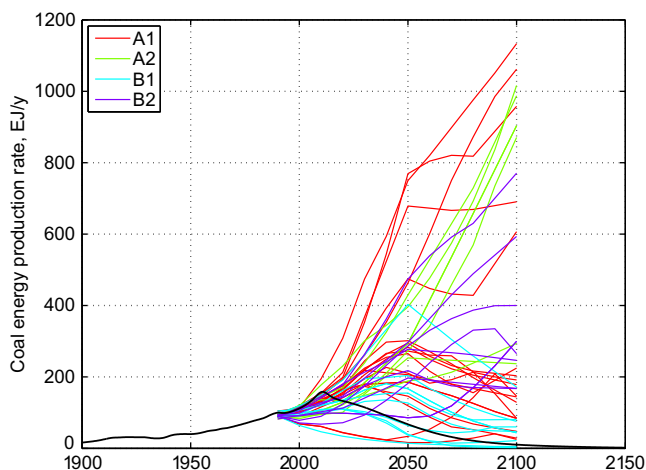
Country	R <sup>2</sup>	MSE <sup>a</sup> (Gt <sup>2</sup> )
China	0.9983	0.1777
USA	0.9998	0.0694
FSU	0.9996	0.0504
Australia	0.9989	0.0052
Former German Empire	0.9999	0.0157
South Africa	0.9998	0.0008
India	0.9992	0.0047
Indonesia	0.9889	0.0005
Mongolia	0.9884	0.0000

<sup>a</sup> MSE = mean squared error =  $(1/n) \sum_{i=1}^n R_i^2$ , where  $R_i$  is the  $i$ th-residual.

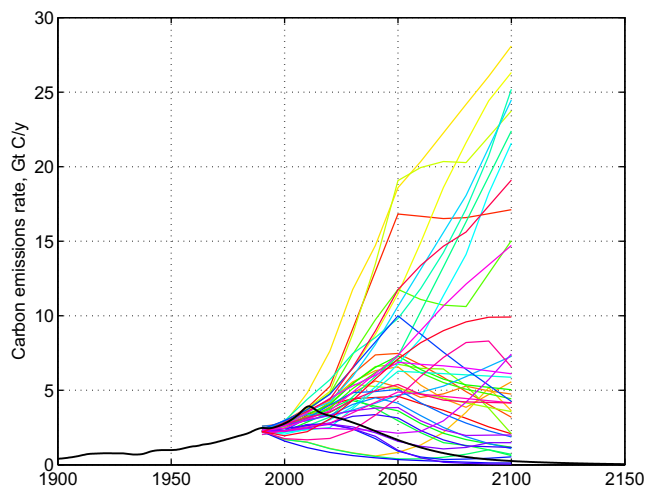
**6. Sensitivity analysis and future work**

Because this study is a multi-cyclic Hubbert analysis, the possibility of future cycles that are not reflected in the historical data must be considered. The base-case in this study includes all coal-producing regions with any significant production history. New mines in existing coalfields should be part of existing Hubbert cycles and thus are part of the base-case. New cycles could occur if a technological breakthrough allowed mining of coal from very thin seams or at much greater depths, or if non-producing coal districts become important producers. Since we do not extract all of any mineral resource, the technology argument will always be there. Looking at recent trends, improvements in coal mining technology have increased the proportion of coal that is extracted, but safety considerations have limited depth. Gradual improvements in recovery percentage should fall within the base-case; only a technology that allows access to a new population of coal seams should create a new fundamental Hubbert cycle, such as in unconventional natural gas recovery in the U.S. [6].

Unlike oil or natural gas, discovery of new fields plays only a minor role in coal resources. Of the major coal districts described in the [Online Supporting Materials](#), only one (Lublin, in Poland) was discovered in the last 50 years. For this reason, the authors contend that the most important quantifiable sensitivities are known, undeveloped resources. Because Alaska’s North Slope and Siberia



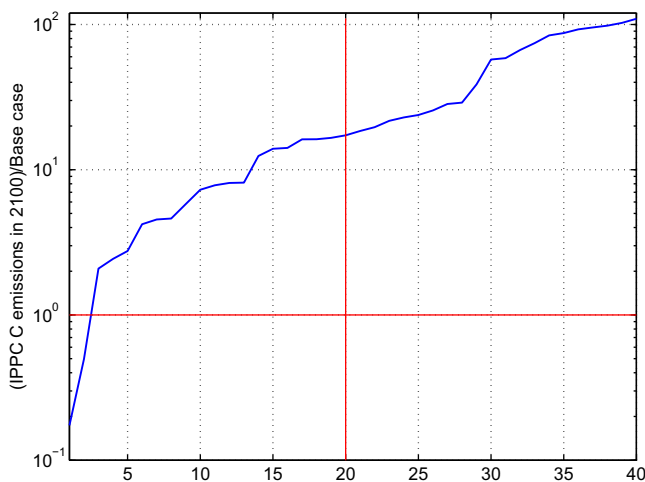
**Fig. 8.** The best multi-Hubbert cycle match of the historical rate of production of energy in coal of all ranks worldwide (thick black line). The 40 IPCC coal energy production scenarios are the thin color curves. Note that most of the IPCC scenarios seem to have little to do with reality predicted by the actual coal production data. In the year 2100, the physical Earth will not be producing 5–7 times more than at the peak in 2011. Data sources: US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), Supplemental Materials to Mohr and Evans (2009) [9], and IPCC (2000) [10].



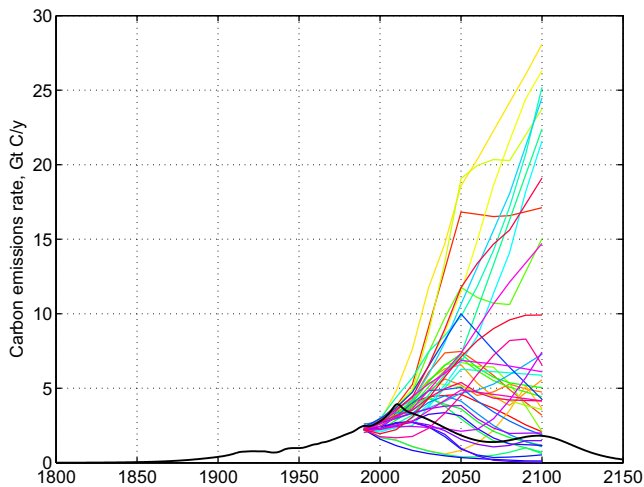
**Fig. 9.** The best multi-Hubbert cycle match of the historical rate of carbon emissions from coal of all ranks worldwide (thick black line). The 40 IPCC coal emissions scenarios are the thin color curves. Data sources US DOE EIA, ([www.eia.doe.gov/fuelcoal.html](http://www.eia.doe.gov/fuelcoal.html)) IEA ([www.iea.org](http://www.iea.org)), Supplemental Materials to Mohr and Evans (2009) [9], and IPCC (2000) [10].

have very large, known, undeveloped coal reserves and because climate change increases access to at least some of these resources, these two areas are likely to dominate any sensitivity analysis of non-producing districts.

Siberia is divided into three political regions; West Siberia, East Siberia and Russian Far East. West Siberia includes the Kuzbass region, described in the [Online Supporting Materials](#), which is Russia’s most important coal-producing region. East Siberia includes the important Kansk-Achinsk soft coal region. Both of these areas have established production and are included in the base-case. The coal resources of the Lena River Valley and part of the Tunguska Basin lie within the Russian Far East. According to Takaishvili and Sokolov [24], the Russian Far East contains 1244 Gt of coal resources, which is 28% of the coal resources of the Russian Federation. The same authors suggest that it would be reasonable to project the ratio of reserves to resources of the entire Russian Federation onto the Russian Far East. For the purpose of this analysis, we will go one step further and project the ratio of reserves to resources from the remainder of the Russian Federation on the Russian Far East. This results in an estimate of 70 Gt for the Russian Far



**Fig. 10.** Comparison of carbon emission predictions in the year 2100. The final predictions of the forty IPCC scenarios are divided by our base-case scenario. For 35/40 scenarios the ratios are above 2, and for 10/40 above 30.



**Fig. 11.** Fig. 9 with an additional recovery of 70 Gt of Siberian coal and 55 Gt of Alaskan coal, both peaking in the year 2100, added in as a sensitivity case.

East. While there is some production in this area already, we assume an entirely new Hubbert cycle with an area of 70 Gt and a peak in the year 2100 at the rate of 1240 million tonnes of coal per year. Applying the ratio of hard coal to soft coal in the A + B + C1 reserves to our 70 Gt estimate gives 28 Gt of hard coal and 42 Gt of soft coal. For the purpose of carbon calculations, it is assumed that the soft coal is subbituminous and the hard coal is bituminous in this case.

The strongest case for a major new coal Hubbert cycle anywhere in the world lies within the United States. The Cretaceous Nanushak Group of Alaska's National Petroleum Reserve (NPR-A) is estimated to contain resources of 1350 Gt of bituminous coal and 1120 Gt of subbituminous coal [25]. The reason that the North Slope coal must be a sensitivity case is there is no recent production history, but this resource could represent up to 10% of the economically recoverable coal in the world [26]. NPR-A area is underlain by permafrost, but so is about 85% of Russia's Pechora Basin [27], which has been an important coal-producing area for decades. The Alaska North Slope coal is low in ash and sulfur content and low in content of elements of environmental concern, such as As, Be, Hg, Mo, Sb and Se [28].

Alaska's coal resources may be undeveloped, but they are not newly-discovered; they were mined for use by whaling ships as early as 1879 [29]. Recent exploration activity has been concentrated in three areas west of the NPR-A, near the Chukchi Sea coast: Cape Beaufort, Deadfall Syncline and Kukpowruk River. Coal beds outcrop around the rims of these three synclines, and extrapolating them across gives total hypothetical resources of 7.2 Gt of coal averaging 28–32 MJ/kg for the three synclines [30]. This area is currently being explored by a joint venture of BHP-Billiton and the Arctic Slope Regional Corporation that drilled 29 coreholes in the coalfields north of Cape Lisburne in 2007–2008, but the corehole results are confidential [31,32].

In their abstract, Sable and Stricker [25] state that the NPR-A may contain as much as one-third of the United States coal resource potential. Since most of the coal is high-quality bituminous, 55 Gt would be a coal resource about half the size of that of the entire U.S. lower-48. As the sensitivity case here, we assume that 55 Gt of the Alaskan coal is produced starting immediately, and the peak of production is in the year 2100. The results of the Alaska sensitivity case are shown in the [Online Supporting Materials](#).

Fig. 11 shows the coal production base-case with these two additional sensitivity cycles superimposed. Note that production of

an additional 125 Gt of coal slows down, but cannot reverse the decline of global CO<sub>2</sub> emissions from coal.

The authors have chosen to include Mongolia in the base-case, but admit that a region with such limited production history represents a test of the Hubbert approach. The question in this case is whether the hard coal mines of the South Gobi Desert represent a separate Hubbert cycle from the soft coal mines of northern Mongolia and, if so, how well the former cycle can be modeled from only 4 years of production history.

Future work should include a similar analysis of liquid petroleum, oil shale, and natural gas. As long as the heavy oil deposits of Canada and Venezuela are treated as sensitivities, a liquid petroleum base-case can be developed with the Hubbert approach. In the case of natural gas, a major challenge will be how to model unconventional gas production from shales, which has been shown to be significant in the USA, but has no production history elsewhere.

## 7. Summary and conclusions

Paraphrasing the words of Chalybäus [3]: “We have discovered here, in the individual as in the general, a law at work, a blind internal necessity, which, however, appears as necessity only because we look at it from without and as objective appearance (phenomenon).” That “objective phenomenon” is the fundamental behavior of large populations of coal mines worldwide and the emerging peak of coal production and CO<sub>2</sub> emissions.

The most important conclusion of this paper is that the peak of global coal production from the existing coalfields is imminent, and coal production from these areas will fall by 50% in the next 40 years. The CO<sub>2</sub> emissions from burning this coal will also decline by 50%. Thus, current focus on carbon capture and geological sequestration may be misplaced. Instead, the global community should be devoting its attention to conservation and increasing efficiency of electrical power generation from coal.

The current paradigms of a highly-integrated global economy and seamless resource substitution will fail in a severely energy-constrained world. A new territory is being charted by all, thus close attention must be paid to what the physical world reveals about energy conservation and production.

Soft coal has a problem akin to that of electric car batteries: it takes a lot of its own energy to move it. Energy content per unit mass of mine-run lignite is about a third that of anthracite. This doesn't matter in the case of lignite production in Poland or Germany because the markets are local power plants. In the U.S., soft coal can be shipped from Powder River Basin because markets are not that far away and the trip is largely downhill (Gillette, Wyoming, is 4800 feet above sea level). Soft coal from Xinjiang has hills to traverse in order to get to markets in China, so it is being developed slowly. Inner Mongolia has better coal, the path from there to markets is downhill and there is large rail capacity from neighboring Shanxi. Siberia is flat and vast so, while hard coal from Kuzbass is shipped by rail, soft coal from Kansk-Achinsk is burned locally in power plants and delivered to markets as electricity.

Carbon dioxide captured from industrial operations can drive new enhanced oil recovery projects and might be injected into coal seams to displace methane. In view of the imminent difficulties with the coal supply, a lasting increase of natural gas production in the United States is of utmost importance. We repeat again that immediate upgrades of the *existing* electrical coal-fired power stations to new, ultra supercritical steam turbines that deliver electrical efficiencies of ca. 50% are urgently needed. The authors do *not* suggest that new coal-fired power plants be constructed, unless they are to replace less-efficient existing coal-

fired plants.<sup>9</sup> The goal should be to increase efficiency rather than capacity.

Scarce coal will make it difficult to justify the energy penalty of CCS [33]; reforestation projects are more useful because they remove CO<sub>2</sub> quickly from the atmosphere, where concentrations will increase for several years after peak carbon emissions.<sup>10</sup> Destruction of tropical forest for biofuel production adds carbon emissions near the peak [35], maximizing damage. Emphasis of a thoughtful greenhouse gas mitigation policy should be on maximizing sequestration rate in the years 2009–2040. Cap-and-trade policies for carbon dioxide emissions will not be effective if the cap is set near peak emission levels, and may allow the natural decline of coal production to effectively subsidize a lack of effort on the part of energy industry.

### Acknowledgements

Greg Croft has been supported for 2 years by the Jane Lewis Fellowship from U.C. Berkeley. We would like to thank Drs. Mohr and Evans for publishing their coal database. We have received critical feedback from Drs. Larry Lake, John Butler, William Spencer, and Sheridan Titman of UT Austin; Richard Norgaard, John Newman and John Prausnitz of UC Berkeley; Sally Benson of Stanford; and Mr. Lucas Patzek, a Ph.D. student at Washington State. Most of their criticisms and suggestions have been incorporated and significantly benefited the paper. We would like to extend special thanks to Dr. Lake for his critique of the initial discussion of emergence of Hubbert cycles. Lucas has edited this paper considerably. Upon reading the paper, Mr. Robert Bryce of Austin has pointed out the remarkable Excel spreadsheet and presentation, *Hubbert's Peak, the Coal Question, and Climate Change*, by Dr. David Rutledge of Caltech. Most of Dr. Rutledge's calculations and conclusions related to coal are very similar to those in this paper. The remaining errors and omissions are the sole responsibility of the authors.

### Appendix A. Why do Hubbert cycles exist?

All quantities whose distributions are considered here are dimensionless.

Soon, it will become obvious that Hubbert cycles are theoretically related to the normal distribution and the error function. However, it is much easier to approximate them with the logistic S-shaped curve and its derivative. The logistic growth curve of human population was first proposed by the Belgian mathematician Pierre François Verhulst after he had read Thomas Malthus' second, expanded edition of the *Essay on the Principle of Population or, a View of its Past and Present Effects on Human Happiness; with an enquiry into our Prospects respecting the Future Removal or Mitigation of the Evils which it occasions*, published in 1803. Malthus was a rare scoundrel [36], but he correctly predicted the ultimate predicament of humanity: too many people competing for insufficient resources on the finite Earth. He also deeply influenced the thinking of David Ricardo, Karl Marx, Charles Darwin, and many other famous economists and scientists, M. King Hubbert being one of them.

<sup>9</sup> "Around 920 U.S. coal plants – 78% of the total – are small (generating less than half a Gigawatt), antiquated and horrendously inefficient. Their average age is 45 years, with many over 75. They tend to be located amidst dense populations and in poor neighborhoods to lethal effect. These ancient plants burn 20% more coal per megawatt hour than modern large coal units and are 60–75% less fuel-efficient than combined cycle gas plants. They account for only 21% of America's electric power but almost half the sector's emissions." *How to End America's Deadly Coal Addiction*, by Robert F. Kennedy, Jr. Published in the Financial Times, July 19, 2009.

<sup>10</sup> The mean residence time of CO<sub>2</sub> in the atmosphere seems to be 20 years [34].

### Appendix A.1. Logistic growth

The cumulative mass production or yield in logistic growth depends on time as follows [37]:

$$m(t) = \frac{m_{\max}}{1 + e^{-r(t-t^*)}} \quad (1)$$

where  $m_{\max}$  is the ultimate production or carrying capacity,  $r$  is the fractional rate of growth;  $t$  is time, and  $t^*$  is the time of maximum production rate:

$$t^* = \frac{1}{r} \ln \left( \frac{m_{\max}}{m(0)} - 1 \right) \quad (2)$$

Note that  $m(0) > 0$ .

The production rate can be obtained by differentiation of the cumulative production  $m(t)$ :

$$\dot{m} = \frac{dm}{dt} = \frac{2\dot{m}^*}{1 + \cosh[r(t-t^*)]} \quad (3)$$

where  $\dot{m}^*$  is the peak production rate.

It may be easily shown that

$$m_{\max} = 4\dot{m}^*/r \quad (4)$$

### Appendix A.2. Logistic versus Gaussian distribution

The logistic Eq. (1) and its derivative, Eq. (2), are close (*but not identical*) to the Gaussian distribution centered at  $t^*$ , and having the standard deviation,  $\sigma$ , related to  $r$  as follows:

$$\sigma \approx \frac{1.76275}{r\sqrt{2 \ln 2}} \quad \text{Matched half – widths} \quad (5)$$

or, better,

$$\sigma \approx \frac{4}{r\sqrt{2\pi}} \quad \text{Matched peaks} \quad (6)$$

See Appendix B for the derivations.

The cumulative logistic growth and its rate then approximate the following equations, theoretically more appropriate for the Hubbert curve models:

$$m(t) \approx \frac{\mathbf{m}_{\max}}{2} \left[ 1 + \operatorname{erf} \left( \frac{(t-t^*)}{\sqrt{2}\sigma} \right) \right] \quad (7)$$

where  $\operatorname{erf}(x) = 2/\sqrt{\pi} \int_0^x e^{-t^2} dt$  is the error function

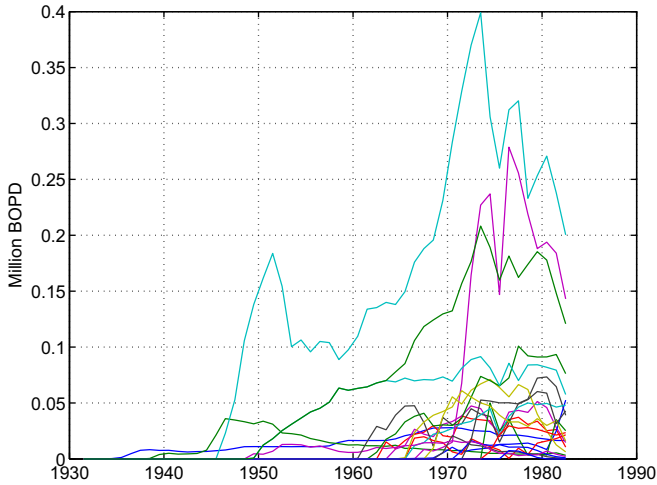
$$\dot{m} = \mathbf{m}_{\max} \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(t-t^*)^2}{2\sigma^2} \right] \quad (8)$$

The two coefficients in bold rescale the normal distribution to the actual production history.

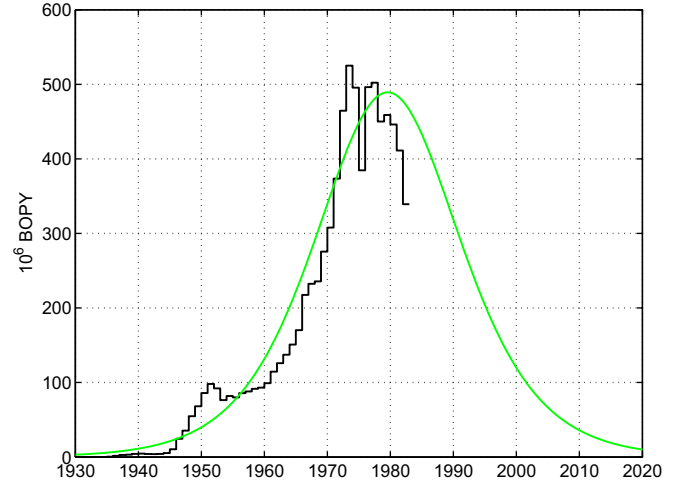
### Appendix A.3. Oilfield examples

An argument advanced against Hubbert has been that the depletion histories of oil reservoirs are distinctly skewed towards longer times. This is true separately for the rate of oil production in every field in a sample, but not so for the sum of these rates. The first example from the Middle East is not perfect because of the complications with data reporting and gradual switching from primary recovery to waterflood. We remind the reader that during





**Fig. 12.** Production from each of the lower Jurassic limestone oilfields in the Middle East can be treated as independent random variable. The total production is then a random-sum ( $\mathcal{R} - S$ ) process that should be a fragment of a normal distribution or a fundamental Hubbert cycle. Data Source: DOE/EIA (1983) [38], Appendix A, Historical Resumé of Oil Production in the Persian Gulf Area.



**Fig. 14.** The total rate of production from the fields shown in Fig. 12 is also fit reasonably well with a single Hubbert curve. The peak production is predicted for 1979, at 500 million barrels of oil per year.

the time period considered in this paper, the Middle East went through three major wars, oil embargo, and the Iranian revolution. We have decided to use the Middle East example – warts-and-all – to illustrate the all too real difficulties in predicting the future.

Here we illustrate the emergence of a Hubbert curve with production data from 23 lower Jurassic limestone oilfields in the Middle East, reported in reference [38], Appendix A. These are: Awali, Damman, Abu Hadriyah, Abqaiq, Qatif, Ghawar, Fadhili, Khursaniyah, Khurais, Harmaliyah, Manifa, Abu Safah, Berri, Sassan, Rostam, Rakhsh, Umm Shaif, Abu al Bakoosh, Dukhan, Idd el Shargi, Maydan Mahzan, Bul Hanine and El Bunduq. The public reporting of most of individual field production stopped by 1982, but the total continued to be reported. We have excluded Ghawar, the largest oilfield on the Earth, because it dominates production from all other fields. However, including Ghawar in the analysis leads to the same conclusions, with the production peak in 1977.

The available historical production of the 23 fields included in the analysis peaked in 1974, and waterfloods were implemented in some of them, resulting in a subsequent total production increase.

Abqaiq was waterflooded starting in the late fifties and Ghawar's three northern culminations were waterflooded starting in the late sixties. Other big waterflood expansions occurred at the two southern culminations of Ghawar (1996), Abu Safah (2004), Qatif (2004), Khursaniyah (2008), and Khurais (2009). The data through 1982 are shown in Fig. 12. Beyond 1982, all we know is that the total production from several of the fields, including Ghawar, started to increase again due to the waterfloods.

We assert that a Hubbert cycle represents a normal process, see e.g., [39,40]. Many variables found in nature result from the summing of numerous unrelated components. When the individual components are sufficiently unrelated, the resulting sum tends towards normality as the number of components comprising the sum becomes increasingly large. Two important conditions for a normal process are: (1) the summation of many discrete (or continuous) random variables, and (2) the independence of these random variables. A normal process is also called a random-sum process ( $\mathcal{R} - S$ -process).

The Central Limit Theorem of statistics states [40] that if  $S_n$  is the sum of  $n$  mutually independent random variables, then the distribution function of  $S_n$  is well approximated by a continuous normal density function, which is given by the formula

$$f(t; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(t - \mu)^2}{2\sigma^2}\right)$$

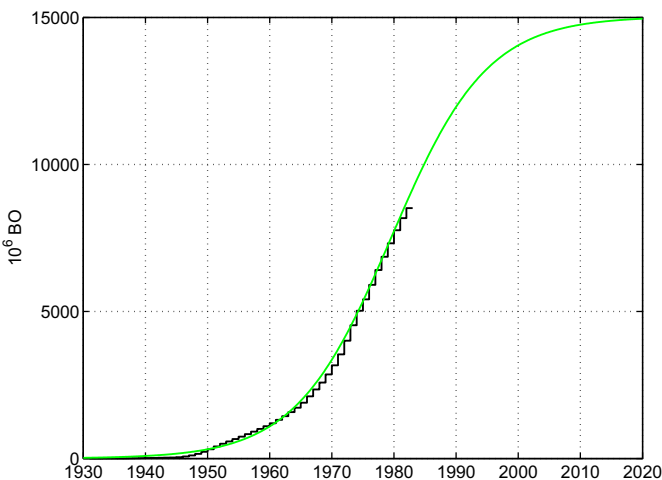
where  $\mu$  and  $\sigma^2$  are the finite mean and variance of the sum. Each variable is sampled at discrete times  $t_j, j = 1, 2, \dots, N$ .

The total *dimensional* production from  $n = 22$  oilfields shown in Fig. 12 should be close to a single Hubbert cycle, described by Eq. (8), or its logistic curve approximation, Eq. (3).

The fitting of the Hubbert cycle to data is done more stably through the cumulative production shown in Fig. 13, but the total rate of production is also fit well, see Fig. 14.

When the Hubbert curve in Fig. 13 is plotted versus the actual production data, Fig. 15, it explains 99% of the variance of the data. However, the residuals of the best Hubbert fit, defined as the cumulative production data minus the Hubbert fit, are correlated, see Fig. 16.

The power spectrum of the numerical Fast Fourier Transform (FFT) of the residuals reveals two dominant frequencies at  $1/23$  and



**Fig. 13.** The total cumulative production from the fields shown in Fig. 12 is fit well with a single Hubbert curve. The ultimate recovery is 15 billion barrels of oil.

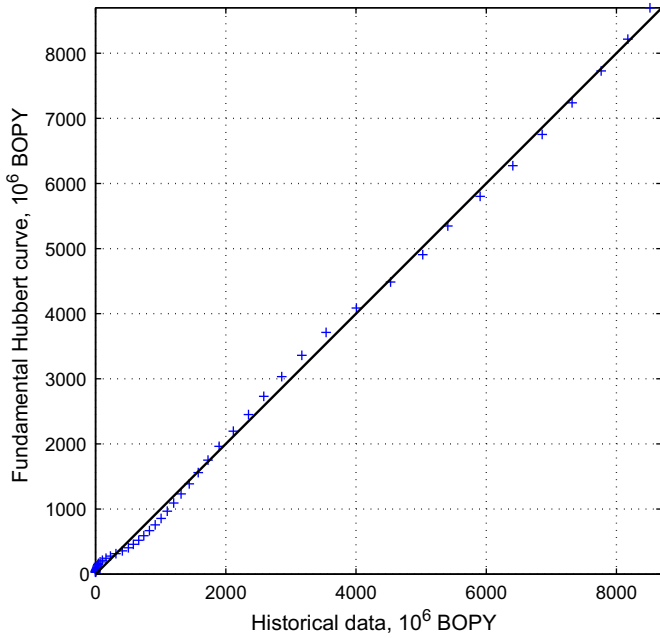


Fig. 15. The cumulative Hubbert curve versus the total cumulative production from the fields shown in Fig. 12. The Hubbert curve explains 99% of the variance of the data.

$1/32 \text{ years}^{-1}$ , see Fig. 17. One may conclude that the characteristic time of oil production from new developments in the same fields is about 27 years. These secondary developments are new random variables, whose sum can be handled with FFT or secondary Hubbert curves, which is the approach chosen in this paper. Either way, importance of the secondary field developments decreases with time, see Fig. 18, and the fundamental Hubbert curve predicts most of oil (or coal or gas) production from a fixed population of deposits.

The multi-Hubbert curve approach in this paper does a good job of capturing the temporary and decaying deviations from the fundamental Hubbert peak, see Figs. 19 and 20. Therefore:

By matching both the cumulative production as well as the rate of production, a multi-Hubbert curve approach may provide a better estimate of the future recovery than a single

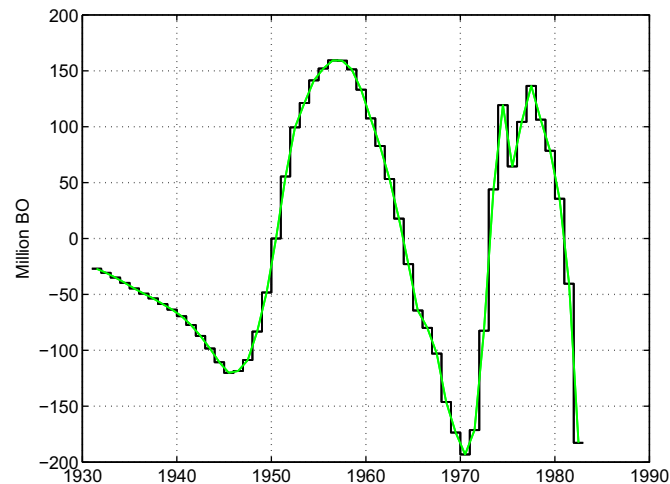


Fig. 16. The Hubbert curve fit residuals (step curve) and their inverse Fast Fourier Transform (FFT) fit (continuous curve).

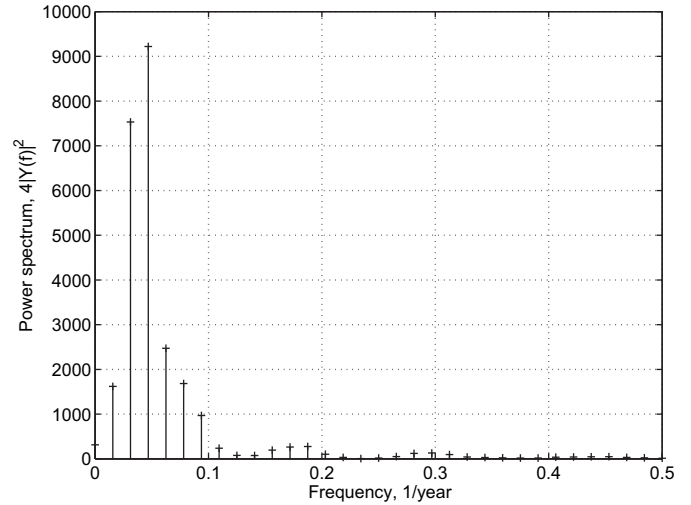


Fig. 17. The power spectrum of the Hubbert curve fit residuals. The dominant frequencies are the inverses of 23 and 32 years. One may conclude that there are ~27-year disturbances that correspond to new field developments. These developments can be handled by adding an inverse FFT-fit of the residuals or by creating separate Hubbert curves.

fundamental Hubbert curve fit of the cumulative production data. In the particular example of the lower Jurassic limestone fields in the Middle East, the ensuing waterflood projects would have to be matched with a separate Hubbert curve peaking much later. We do not have the field-specific data beyond 1982 to quantify this assertion.

The second example consists of 65 offshore oilfields in Norway, whose production rates are shown in Fig. 21. Out of the tangled spaghetti of individual field production curves, there emerges a picture-perfect normal distribution in Fig. 22. The cumulative production is matched perfectly and the predicted ultimate oil recovery is 26 billion barrels. A two-curve approximation is marginally better, see Fig. 23, but it does not matter. The dominant frequencies of the new field projects or project expansions are about  $1/13$  and  $1/30 \text{ years}^{-1}$ .

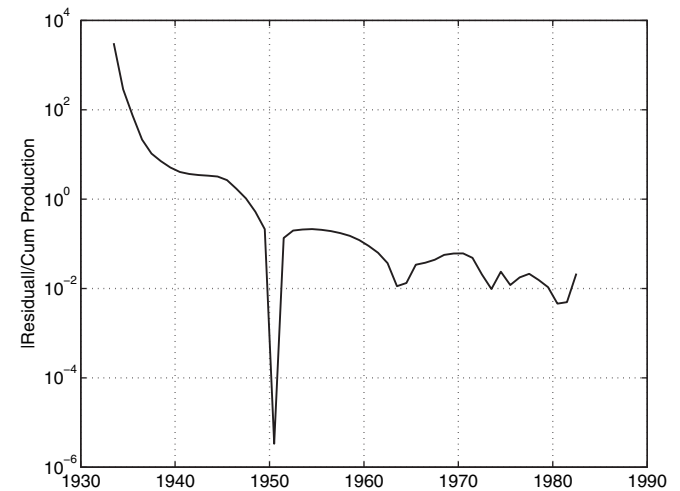
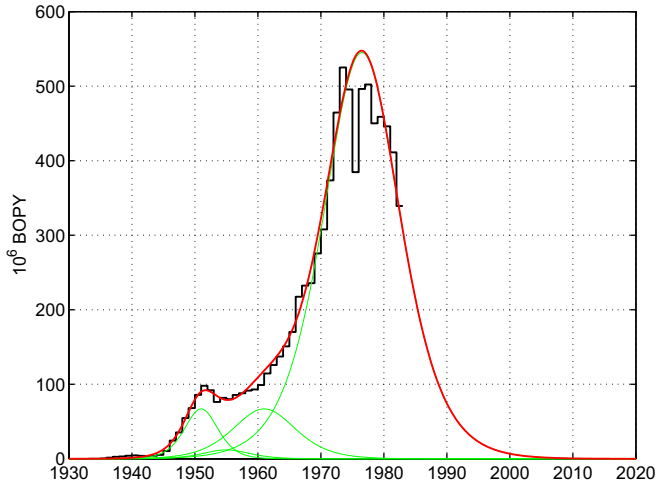
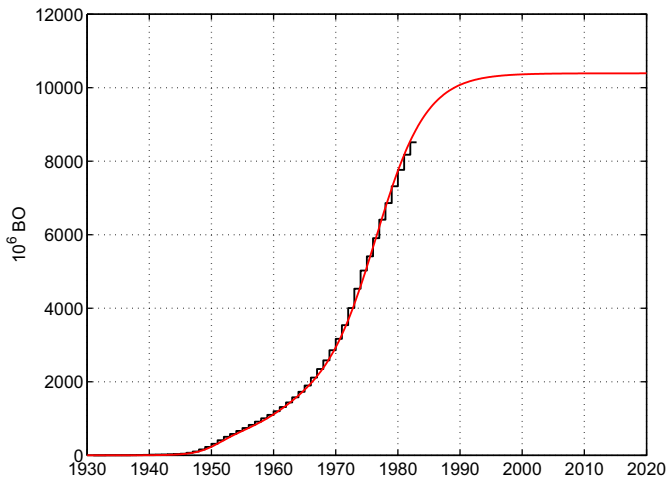


Fig. 18. The Hubbert peak residuals (new projects) divided by the elapsed cumulative production are strongly damped. They do not follow a single frequency, but damping is approximately exponential. This means that in the long run a fundamental Hubbert peak predicts most of oil production from a population of oilfields it tracks.



**Fig. 19.** The multi-Hubbert curve approach used in this paper predicts oil production peak in 1976 at 545 million BO per year.



**Fig. 20.** The multi-Hubbert curve approach fits cumulative oil production very well, predicting 10.4 billion barrels of ultimate recovery from primary production and early waterfloods, 70% of the ultimate recovery predicted in Fig. 13. As far as we can tell, this prediction happens to be closer to the actual production history prior to the full waterflood implementation. The waterfloods would have to be modeled with a separate Hubbert curve.

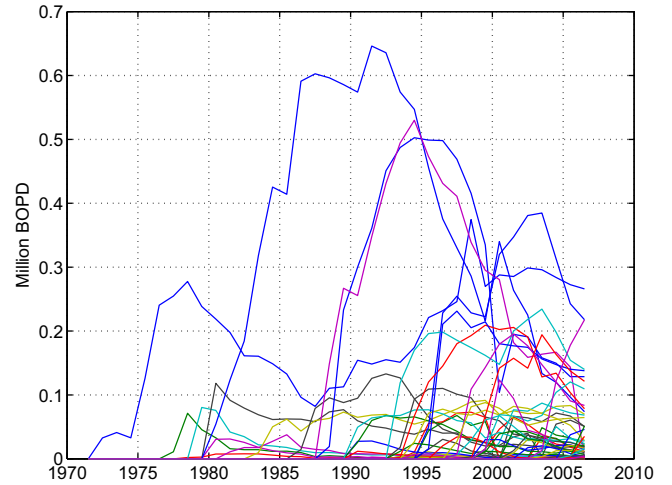
Norway has been a unique human experiment; it had no wars, revolutions, nor Great Depressions over the time period of the North Sea development. What did vary, and wildly, was the price of oil. That variation apparently had no effect on curtailing oil production and distorting the Hubbert cycle. Mixing Norwegian Sea and North Sea production also was not a problem; these are two different areas, but similar geology and operating conditions caused them to act as a single cycle. The most serious deviation between the oil production and the Hubbert cycle was caused by the very low oil prices between 1995 and 2000.

**Appendix B. Matching the logistic and normal distributions**

To approximate the Gaussian distribution (rate) curve,

$$f_1(t; \sigma, t^*, m_{\max}) = m_{\max} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(t - t^*)^2}{2\sigma^2}\right] \quad (9)$$

with the logistic curve,



**Fig. 21.** Production from each of the 65 North Sea and Norwegian Sea oilfields in Norway can be treated as an independent random variable. The total production is then a random-sum ( $\mathcal{X} - S$ ) process and it should yield a Gaussian distribution. The data are from Pennwell Publishing Co.: The International Encyclopedia (1970–1984) and Oil & Gas J. (year-end issues, 1985–2007).

$$f_2(t; r, t^*, m_{\max}) = \frac{m_{\max} r}{2} \frac{1}{1 + \cos h[r(t - t^*)]} \quad (10)$$

we will first attempt to match their full widths at 1/2 heights. For the Gaussian distribution

$$f_1(t_0; \sigma, t^*, m_{\max}) = \frac{1}{2} f_1(t^*; \sigma, t^*, m_{\max})$$

$$\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(t_0 - t^*)^2}{2\sigma^2}\right] = \frac{1}{2} \frac{1}{\sqrt{2\pi}\sigma} \quad (11)$$

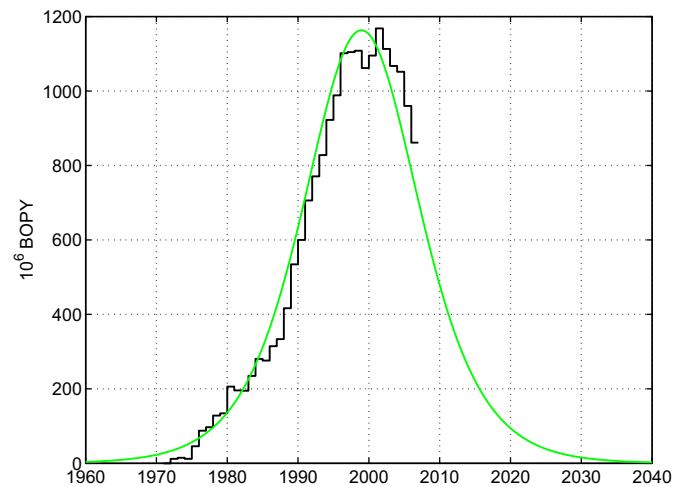
$$\exp\left[-\frac{(t_0 - t^*)^2}{2\sigma^2}\right] = \frac{1}{2}$$

and

$$\frac{(t_0 - t^*)^2}{2\sigma^2} = \ln 2$$

$$t_0 = \pm \sigma \sqrt{2 \ln 2} + t^* \quad (12)$$

The full width at half maximum is therefore



**Fig. 22.** The total rate of production from the fields shown in Fig. 21 is an almost perfect Hubbert curve. The peak production is predicted for the year 1999, at 1170 million barrels of oil per year.

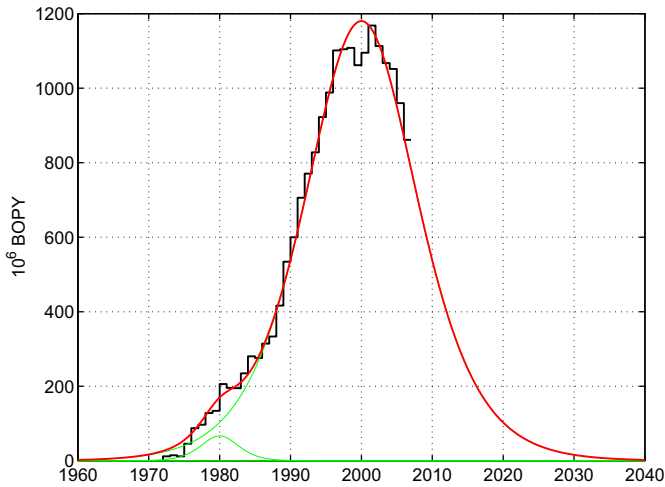


Fig. 23. The multi-Hubbert curve approach used in this paper predicts oil production peak in 2000.5 at 1180 million BO per year.

$$2\sigma\sqrt{2 \ln 2}$$

for the Gaussian distribution.

For the logistic distribution

$$f_2(t_0; r, t^*, m_{\max}) = \frac{1}{2}f_2(t^*; r, t^*, m_{\max})$$

$$\frac{m_{\max}r}{2} \frac{1}{1 + \cosh[r(t_0 - t^*)]} = \frac{m_{\max}r}{8} \tag{13}$$

$$\frac{1}{1 + \cosh[r(t_0 - t^*)]} = \frac{1}{4}$$

$$\cosh[r(t_0 - t^*)] = 3$$

An approximate solution to this transcendental equation is  $r(t_0 - t^*) \approx 1.76275$ . The full width at half maximum is therefore

$$\frac{3.5255}{r}$$

for the logistic distribution.

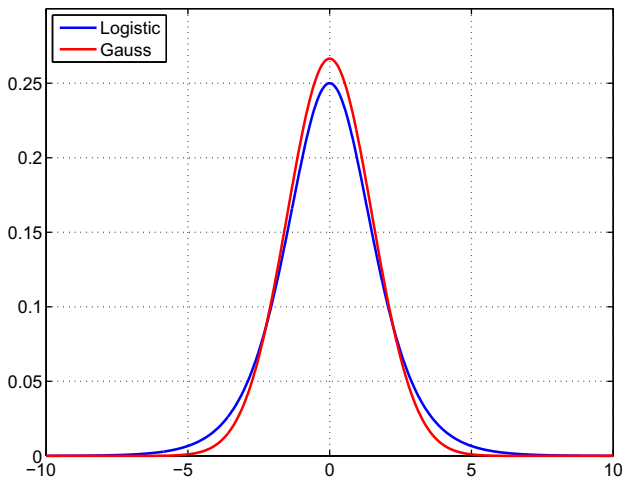


Fig. 24. Logistic versus normal distribution with matched half-widths, for  $r = 1$ ,  $t^* = 0$ ,  $m^* = 1/4$ .

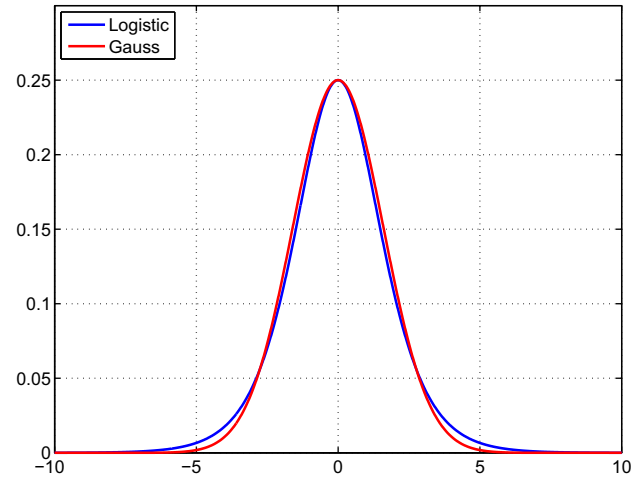


Fig. 25. Logistic versus normal distribution with matched peaks, for  $r = 1$ ,  $t^* = 0$ ,  $m^* = 1/4$ .

To match the respective full widths, we require that

$$2\sigma\sqrt{2 \ln 2} = \frac{3.5255}{r} \tag{14}$$

$$\sigma = \frac{1.76275}{r\sqrt{2 \ln 2}}$$

To match the peaks, we require that

$$\frac{m_{\max}}{\sqrt{2\pi}\sigma} = \frac{rm_{\max}}{4} \tag{15}$$

$$\sigma = \frac{4}{r\sqrt{2\pi}}$$

Since heights of the two distributions are somewhat different for equal half-widths (the  $t_0$  in Eq. (13)<sub>4</sub> is in fact different than that in Eq. (11)<sub>3</sub>), the half-width match is approximate, see Fig. 24. The peak matching is also approximate, see Fig. 25, but seems to be better.

### Appendix C. Heating values and CO<sub>2</sub> emissions for coal

The coal heating value data, expressed as the HHVs of coals of different ranks, are listed in Tables 3 and 4. The weighted averages of the data rounded off to 2 significant digits are shown in column 1 of Table 5.

The measured CO<sub>2</sub> emissions from coals of different ranks, reported in [41] and listed in Table 5, are well approximated by the following equation:

$$\frac{\text{kg CO}_2}{\text{kg coal}} \approx \frac{\text{HHV of coal}}{32.8 \times 1.22} \tag{16}$$

see Fig. 26. Here the enthalpy of oxidation of the solid carbon, 32.8 MJ/kg, see Tables 5–76 in [42], is adjusted upwards for the heating effects of hydrogen in the coal. Eq. (16) allows for an easy conversion of coal HHV to CO<sub>2</sub> emissions. The correlation between the coal HHV and its CO<sub>2</sub> emissions should be quite accurate for most major coal-producing countries.

The “low value coals” have high ash and/or moisture content and/or high sulfur content; their HHVs generally are below 17 MJ/kg. About one-third of world coal production falls into this category [43]. The top 6 countries producing such coals are India (250 million tonnes per year), Germany (187), Russian Federation (100), USA (80, lignite only), Australia (74), and Poland (64).

Even though the HHVs of the Chinese bituminous coals listed in Table 3 are in the range of 25–30 MJ/kg, the average raw

**Table 3**  
Heating values of world coals.

Coal name	LHV (Btu/lb)	LHV (kcal/kg)	HHV <sup>a</sup> (MJ/kg)	Source
<i>Anthracite</i>				
Pennsylvania #4 premium	13,200		32.8	Lehigh Coal
Pennsylvania #4 standard	12,500		31.1	Lehigh Coal
China Shenrong 1	7000		31.4	Shenrong Carbon Ltd.
China Shenrong 2	7100		31.8	Shenrong Carbon Ltd.
<i>Bituminous coal</i>				
N. Appalachian	13,000		32.3	EIA
C. Appalachian	12,500		31.1	EIA
Illinois Basin	11,800		29.3	EIA
Uintah Basin	11,700		29.1	EIA
Penn Keystone low ash	12,700		31.6	Penn Keystone Coal Co.
Penn Keystone High Vol.	11,500		28.6	Penn Keystone Coal Co.
Penn Keystone Low Vol.	12,000		29.8	Penn Keystone Coal Co.
China Jining 3-1			29.5	[45]
China Jining 3-2			28.6	[45]
China Zibo average			27.6	[46]
China Banshan			25.0	[47]
China Datong			30.5	[27]
China Huabei			29.5	[27]
China Antaibao			29.5	[27]
China Average Coal			23.7	[44]
Indonesia BCI 5300		5300	23.7	Borneo Coal Indonesia
Indonesia BCI 5500		5500	24.6	Borneo Coal Indonesia
Indonesia BCI 5800		5800	26.0	Borneo Coal Indonesia
Indonesia BCI 6000		6000	26.9	Borneo Coal Indonesia
Indonesia BCI 6200B		6200	27.8	Borneo Coal Indonesia
Indonesia steam		5800	26.0	Daryalal Trading
South Africa steam		6700	30.0	Daryalal Trading
Australia thermal		6700	30.0	Centennial Coal
<i>Subbituminous coal</i>				
PRB Wyodak	8220		20.4	[48]
<i>Lignite coal</i>				
Texas Wilcox	6500		16.2 <sup>b</sup>	[49]

<sup>a</sup> 1.07 of the LHV after the unit conversions [50].

<sup>b</sup> Presumably dry lignite. Lignite at the power station gate will be moist and have a lower heating value.

**Table 4**  
Platt's coal specifications [51].

Coal name	LHV (Btu/lb)	HHV <sup>a</sup> (MJ/kg)
Pittsburg Seam <sup>b</sup>	13,000	32.4
Upper Ohio River	12,500	31.1
Big Sandy River	12,500	31.1
Thacker/Kenova	12,500	31.1
Illinois Basin 1	11,800	29.4
Illinois Basin 3	11,000	27.4
Illinois Basin 4	10,500	26.1
Powder River Basin 1	8800	21.9
Powder River Basin 2	8400	20.9
Colorado 1	11,700	29.1
Colorado 2	11,000	27.4
Colombia Bolivar 1 <sup>c</sup>	11,600	28.9
Colombia Bolivar 2	11,300	28.1
Poland Baltic	11,300	28.1
S. Africa Richards Bay	11,200	27.9
Australia Gladstone	11,700	29.1
China Qinhuangdao	11,200	27.9
Indonesia Kalimantan	9000	22.4

<sup>a</sup> 1.07 of the LHV after the unit conversions [50].

<sup>b</sup> Source: Platt's Weekly U.S. Price Survey, [www.platts.com/CommodityHome.aspx?Commodity=Coal](http://www.platts.com/CommodityHome.aspx?Commodity=Coal).

<sup>c</sup> Source: Platt's International Coal Report, [www.platts.com/CommodityHome.aspx?Commodity=Coal](http://www.platts.com/CommodityHome.aspx?Commodity=Coal).

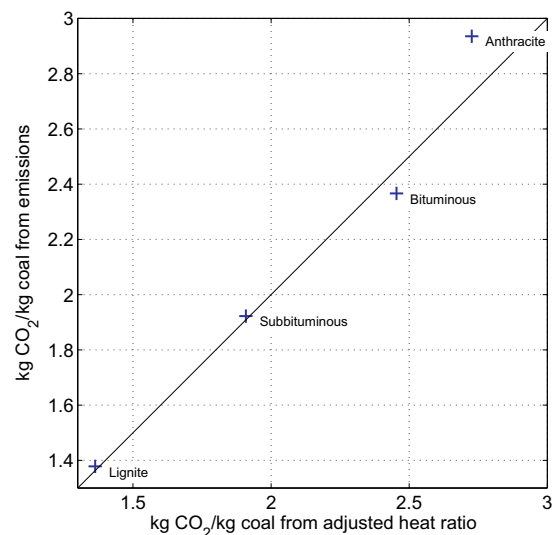
Chinese coal has the HHV of only 24 MJ/kg [44].<sup>11</sup> The raw Polish bituminous coals have the HHVs between 20 and 28 MJ/kg, with the average value of 26 MJ/kg guaranteed in one particular

**Table 5**  
Summary of coal heating values and specific CO<sub>2</sub> emissions.

Rank	HHV <sup>a</sup> (MJ/kg)	lb CO <sub>2</sub> /MBtu <sup>b</sup>	g CO <sub>2</sub> /MJ	kg CO <sub>2</sub> /kg coal
Anthracite	30	227.4	97.9	2.936
Bituminous	27	203.7	87.7	2.367
Subbituminous	21	212.7	91.5	1.922
Lignite	15	213.5	91.9	1.378

<sup>a</sup> HHV = Higher Heating Value.

<sup>b</sup> From [41]



**Fig. 26.** A fit of the measured CO<sub>2</sub> emissions with Eq. (16). The fit is least good for anthracite, but very little of anthracite is produced worldwide nowadays.

<sup>11</sup> Assuming the average coal heating value reported in [44] is lower (LHV).

contract.<sup>12</sup> This value is below the 27 MJ/kg assumed for an average bituminous coal.

## Appendix. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.energy.2010.02.009.

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<sup>12</sup> Private communication from Mr. Wieslaw Krawet, based on the initial specifications of a contract to build a new power generation block in Siekierki, Poland.